RESEARCH PAPER P-351

DESIRABLE ILLUMINATIONS FOR CIRCULAR APERTURE ARRAYS

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400 Army-Navy Drive, Arlington, Virginia 22202

Contract DAHC15 67 C 0011
ARPA Assignment 5

ABSTRACT

The design of phased-array antennas has progressed to the point where side-lobe control is limited by practical problems of aperture efficiency and element tolerances. So far as the theoretical limits are concerned it is possible to provide a variety of designs meeting any side-lobe specification desired. This being the case, it becomes desirable to examine the requirements for low side lobes more critically. In particular, it is felt that more attention should be given to desirable relations between the average side-lobe level and peak side-lobe level.

To provide some insight into the type of pattern control that is possible, numerical calculations have been made for a variety of illuminations having the same aperture efficiency. It is shown that by allowing the peak side lobe to increase slightly, important improvements can be made in the level of the more remote side lobes.

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I. INTRODUCTION

With the advent of modern synthesis techniques and the widespread use of phased arrays, it is now possible to design antennas to have any desired side-lobe level. With the older reflector-type antennas, side-lobe suppression was limited by the fact that only limited control of the illumination function was available. With corporate-fed arrays, however, it is possible to adjust the excitation for each element individually, enabling a close approximation of any desired illumination function. Even with optically fed systems, we can obtain considerable control of the illumination through the use of multihorn feeds. As our technology improves, however, it becomes desirable to examine our requirements more critically. If we continue to specify the maximum side-lobe suppression that is considered achievable, we may in some cases pay undue penalties with regard to beamwidth and gain.

Among the characteristics that could profitably receive more attention is the question of the balance between close-in side lobes versus more remote side lobes. Using the well-known Taylor synthesis,[1,2] we have a choice of the parameter $\bar{n}$. If we choose a large value of $\bar{n}$, we can lower the peak side-lobe level while maintaining good efficiency. We do this, however, at the cost of having a large number of side lobes whose level is very nearly the peak side-lobe level. To maintain the same aperture efficiency with smaller values of $\bar{n}$, we must accept a somewhat higher level for the close-in side lobes, but the more remote side lobes will be reduced. An alternative method of controlling the more remote side lobes is the use of synthesis techniques based on the Bickmore-Spellmire[3] two-parameter family of illuminations.

In order that these alternative techniques may be evaluated more clearly, we have made numerical pattern computations for a number of different illuminations, all having the same normalized beamwidth. In particular, we have constrained the illuminations to yield a 1-deg beamwidth for an aperture 70 wavelengths in diameter.

II. TAYLOR ILLUMINATIONS

For antennas in which it is possible to control the illumination to any desired function, the most common design procedure is based on the work of T.T. Taylor[1]. This work is in turn based on the prior work of C.L. Dolph[2].

Dolph showed that for a uniformly spaced linear array, the minimum side-lobe level for a given beamwidth is obtained when the illumination is adjusted to yield a radiation pattern of the form

$$f(\theta) = k \cos\left\{(N-1)\frac{\cos\frac{\varphi}{2}}{\cos\frac{\varphi_o}{2}}\right\} = K \cos\{(N-1)\psi\}$$

where,

N is the number of elements in the array

$\varphi = 2\pi\frac{d}{\lambda}\sin\theta$ is the phase advance per element

ϕ_o is a parameter of the illumination

θ is the angle from the array normal

d is the spacing between elements

$\psi = \cos^{-1}\left\{\frac{\cos\frac{\varphi}{2}}{\cos\frac{\varphi_o}{2}}\right\}$ is an auxilliary variable

We note that when $\varphi_o < \varphi < 2\pi - \varphi_o$ then,

$$\left|\frac{\cos\frac{\psi}{2}}{\cos\frac{\varphi_o}{2}}\right| < 1$$

and ψ is real. In this range, the cosine term of $f(\theta)$ oscillates between 1 and -1. On the other hand, when $|\varphi| < \varphi_o$ then ψ becomes imaginary and we have

$$f(\upsilon) = K \cosh\left((N-1)\Gamma\right),$$

where,

$$\cosh \Gamma = \frac{\cos \frac{\psi}{2}}{\cos \frac{\varphi_0}{2}} .$$

By a suitable choice of φ_0, we can make the ratio of main-lobe gain to side-lobe gain as large as we like. The price we pay for a very low side lobe is a broadening of the main beam and a loss of gain. The design is then a compromise between beamwidth and side-lobe requirements.

Extending this result to the case of a continuous circular aperture, Taylor devised a set of illuminations in which the radiation pattern approximates the equal ripple characteristic of the Dolph-Tchebycheff array.* Exact reproduction of the equal ripple characteristic would require a singular illumination function which is not realizable with finite energy. Taylor avoided the singularities by allowing the beamwidth to broaden slightly and approximating the equal ripple characteristic over only a finite number of the side lobes. The function he actually approximates is

$$f_1(\theta) \approx \cos\left(\pi \sqrt{(u/\sigma)^2 - A^2}\right)$$

where,

$u = (D/\lambda) \sin \theta$

σ is a dilation factor

A is the parameter setting the side-lobe level.

*The continuous distribution can be closely approximated by a discrete planar array if the number of elements is large. The discrete array does introduce grating lobes that affect the far out side-lobe level, especially when the array is steered well off boresight. This effect is controlled by regulating the interelement spacing.

The technique used is to adjust the dilation factor σ so that the $\bar{n}^{th}$ null of the desired pattern coincides with the $\bar{n}^{th}$ null of a uniformly illuminated circular aperture. The illumination is then adjusted so that for $n \leq \bar{n}$ the n^{th} null of the actual pattern coincides with the n^{th} null of the desired pattern while for $n \geq \bar{n}$, the nulls correspond to those of the uniformly illuminated aperture. Taylor illuminations then depend on two parameters $\bar{n}$ and A; A sets the side-lobe level and $\bar{n}$ determines how many side lobes have the null spacings of the equal ripple pattern.

As in the case of the Dolph-Tchebycheff array, Taylor illuminations can be designed for any desired side-lobe level. As the side-lobe suppression is increased, the beamwidth for a given aperture broadens so that again the design involves a compromise between beamwidth and side-lobe level.

In contrast to the Dolph-Tchebycheff illuminations, the Taylor illuminations have a second parameter $\bar{n}$ which the designer may vary. As $\bar{n}$ is increased, the accuracy of approximation to the equal ripple characteristic improves. For a given side-lobe level, the beamwidth narrows with increasing $\bar{n}$. On the other hand, if the beamwidth is held constant the side-lobe level can be improved by raising $\bar{n}$. On superficial inspection one would be inclined to use very large values of $\bar{n}$. One disadvantage of doing this is that the resulting illuminations become more rapidly varying and therefore more difficult to synthesize. If the illumination varies rapidly from element to element, the mutual coupling effects will disturb the illumination. Also the discrete array is then a less accurate approximation of the continuous aperture. Perhaps a more important disadvantage of large values of $\bar{n}$ is the fact that although the peak side-lobe level is reduced, there are more side lobes having amplitudes near the peak and the amplitude of the more remote side lobes is increased with increasing $\bar{n}$.

Figure 1 is a plot of normalized beamwidth versus side-lobe level for Taylor illuminations having various values of $\bar{n}$. There is a limitation on the minimum value of $\bar{n}$ that may be used. If $\bar{n}$ is too small, the dominant side lobe will not be in the controlled part of the pattern. As a practical matter $\bar{n}$ should be 3 or greater for side-lobe levels from 15 to 33 db and at least 4 for side-lobe levels greater than 34 db.

To illustrate the effect of $\bar{n}$ on the shape of the side-lobe envelope, we limit our attention in what follows to illuminations having the same normalized beamwidth of 70 deg λ/D. In other words, we limit ourselves to illuminations which will produce a 1-deg beam from an aperture 70 wavelengths in diameter. Figure 2 is a plot of peak side-lobe level as a function of $\bar{n}$ for this limitation. Figure 3 shows the actual illumination functions for $\bar{n}$ = 3, 7, 10, and 15. Also shown on this illustration are the illuminations for two of the Bickmore-Spellmire patterns which will be discussed more fully in the next section.

Figure 4 shows the radiation patterns corresponding to the illuminations of Fig. 3, and Fig. 5 shows the side-lobe envelopes superimposed. It is readily apparent that in the more remote regions, the side-lobe level increases with increasing $\bar{n}$ although the reverse is true for the maximum side lobe.

III. BICKMORE-SPELLMIRE PATTERNS

Bickmore and Spellmire[3] have discussed a two-parameter family of radiation patterns which includes most of the more commonly synthesized patterns as special cases.* The patterns are conveniently expressed in terms of lambda functions

$$f(\theta) = \Lambda_\nu \left(\pi\sqrt{u^2 - A^2}\right)$$

Where

$$\Lambda_\nu(x) = \Gamma(\nu + 1)\left(\frac{2}{x}\right)^\nu J_\nu(x)$$

$u = D/\lambda \sin\theta$

A = a parameter setting the general side-lobe level

ν = a parameter setting the rate at which successive side lobes decrease

The lambda functions are not as familiar as they should be. A tabulation for integer values of ν is given in Jahnke and Emde[4] and for values of ν which are half an odd integer the lambda functions can be conveniently expressed in terms of circular functions. In particular

$$\Lambda_{-\frac{1}{2}}(x) = \cos x$$

$$\Lambda_{\frac{1}{2}}(x) = \sin x/x$$

$$\Lambda_{\frac{3}{2}}(x) = 3\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right)$$

*In their paper, Bickmore and Spellmire treated only the continuous line source. As we shall see, however, the patterns are also realizable with circular apertures.

Figure 6 is a plot of peak side-lobe ratio and beamwidth for various values of A and ν. Figure 7 shows how successive side lobes fall off more rapidly as ν is increased. Asymptotically the more remote side lobes fall off as distance to the $\nu + \frac{1}{2}$ power. These two figures enable one to determine the given characteristics of a particular Bickmore-Spellmire design.

If $\nu \geq \frac{1}{2}$, the Bickmore-Spellmire pattern can be realized with an analytic illumination on a line source while if $\nu \geq 1$ they can also be realized with an analytic illumination on a circular aperture. For the line source, the appropriate illumination function is of the form

$$G(x) = \left(1-x^2\right)^{\nu-\frac{1}{2}} \Lambda_{\nu-\frac{1}{2}} \left(\pi A \sqrt{x^2-1}\right),$$

where G is the illumination amplitude at position x on a line source extending from -1 to 1. For the circular aperture, the appropriate illumination function is of the form

$$G(\rho) = \left(1-\rho^2\right)^{\nu-1} \Lambda_{\nu-1} \left(\pi A \sqrt{\rho^2-1}\right),$$

where G is the amplitude at a radius ρ of an aperture whose maximum radius is 1. When $\nu < \frac{1}{2}$ for the line source or $\nu < 1$ for the circular aperture, then the Bickmore-Spellmire patterns can still be approximated with analytic illuminations by using Taylor's technique of matching the first $\bar{n}$ zeros of the actual pattern to those of the pattern being approximated. The Taylor illuminations are in fact such an approximation for the case of $\nu = -\frac{1}{2}$. For the present, however, we limit ourselves to the cases $\nu = 1$ and $\nu = 2$ for which approximations are not necessary.

Figures 3e and f show the illuminations for the Bickmore-Spellmire patterns of $\nu = 1$ and $\nu = 2$ constrained to have a normalized beamwidth of 70 deg λ/D. The radiation patterns are shown in Fig. 4 and the side-lobe envelopes in Fig. 5.

IV. EVALUATION OF PATTERNS

Examination of Figs. 4 and 5 shows that if we relax the requirement on the peak side-lobe level, we can achieve important improvements of the more remote side lobes. In fact, at a distance of 7 beamwidths or more, the order of superiority is just the reverse of that based on the maximum side lobe. The question of which pattern is best for any given situation requires rather critical examination of the requirements. Unfortunately, we seem to be raising a problem without providing a solution.

If the only interference source were a single jammer known to be, say, 20 db stronger than the desired signal, and if 17.5 db signal-to-jam ratio were required for satisfactory operation, then the best pattern would be the one which provided at least 37.5 db side-lobe rejection over the greatest possible fraction of the sky. This is the type of application for which the Dolph-Tchebycheff philosophy is appropriate and, of the patterns illustrated, the Taylor $\bar{n}$ = 15 is the nearest to optimum. In a more general situation, where there are several sources of interference and the interference level is variable, we tend toward a criterion based on average rather than peak side-lobe level. If we take as our criterion of optimization the minimization of the total power radiated outside the main beam, then the Taylor design with $\bar{n}$ = 7 is superior to either $\bar{n}$ = 3 or $\bar{n}$ = 10 and the Bickmore-Spellmire design for ν = 1 is superior to any of the Taylor designs. This conclusion is predicated on the beamwidth being narrow so that most of the side-lobe power lies in the visible space.

An additional factor which should enter into the design process is the question of tolerances. If we assume that each element of an array radiates two components, one the design value and the other an independently random perturbation, then the pattern of the array will be the superposition of the design pattern and a random error pattern. The expected power of the random pattern will depend both on the number of elements and on the element tolerances. For arrays having relatively few elements, it would not be worthwhile to design for very low side lobes unless the tolerances can be maintained very precisely. In this case, a balanced design might be achieved by choosing a Taylor illumination with a design side-lobe level about equal to the expected random lobe level. For a very large number of elements, on the other hand, it may be that such a balance would result in side lobes that are lower than necessary; in this case, we would be paying too high a price in aperture efficiency so one of the Bickmore-Spellmire designs would be preferable.

V. CONCLUSIONS

Unfortunately, we are not able to write a prescription for the optimum design to fit a general situation. We do, however, wish to point out that the problem is somewhat more complex than commonly assumed. As our technology improves and it becomes possible to design for lower and lower side lobes, it becomes imperative that we provide more careful consideration of the side-lobe requirements. It appears that more attention to the relative amplitude of the remote and close-in side lobes is warranted. In particular, we tend to question the use of large values of $\bar{n}$ in Taylor's illuminations and we favor consideration of the Bickmore-Spellmire illuminations when tolerances warrant. We might also mention that because the Bickmore-Spellmire illuminations are smoother they can be more easily realized than the Taylor illuminations which have high values of $\bar{n}$. This is especially true when we are using multihorn optical feeds.

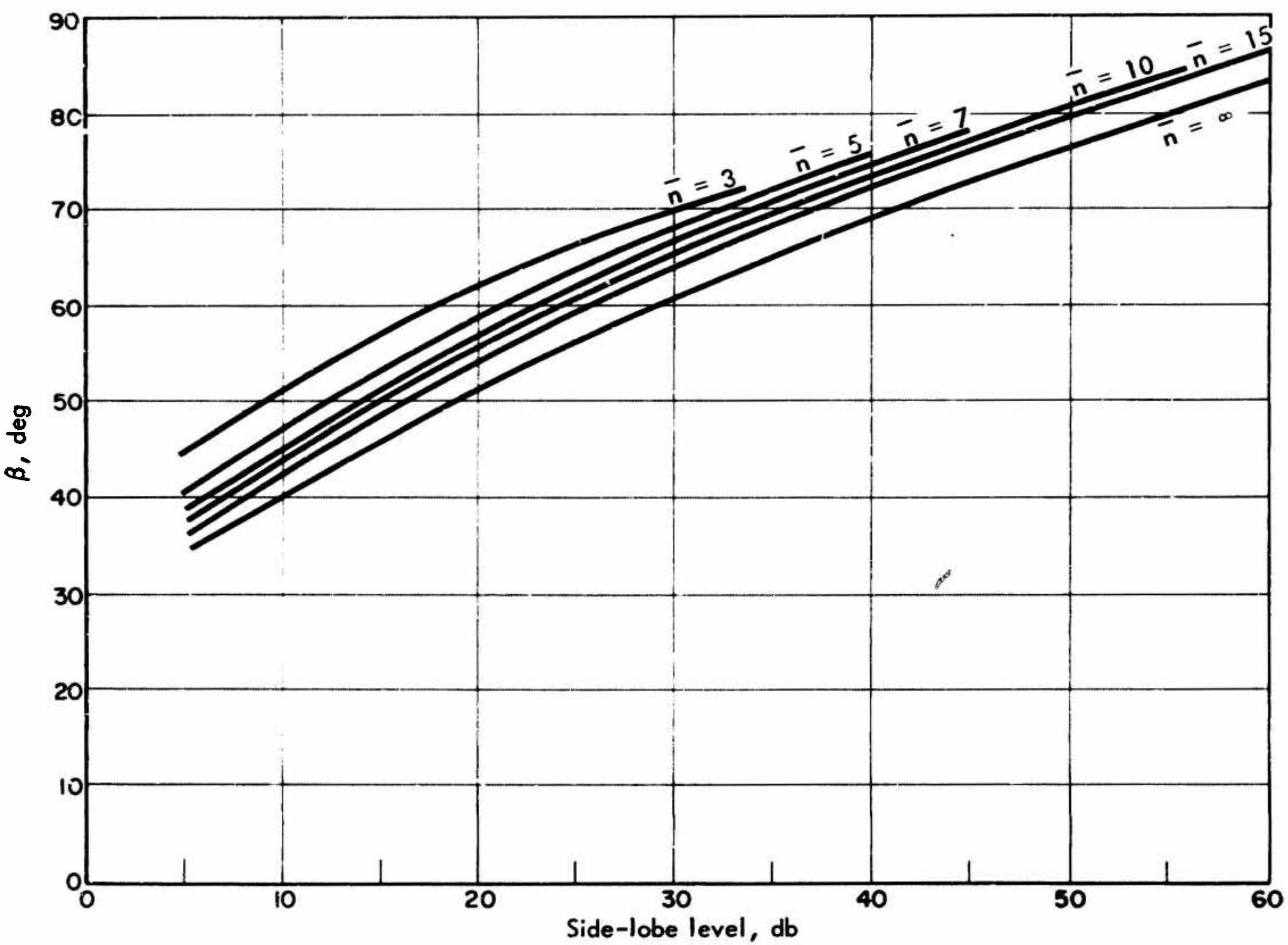

FIGURE 1 Beamwidth Versus Side-Lobe Level, Taylor Illumination

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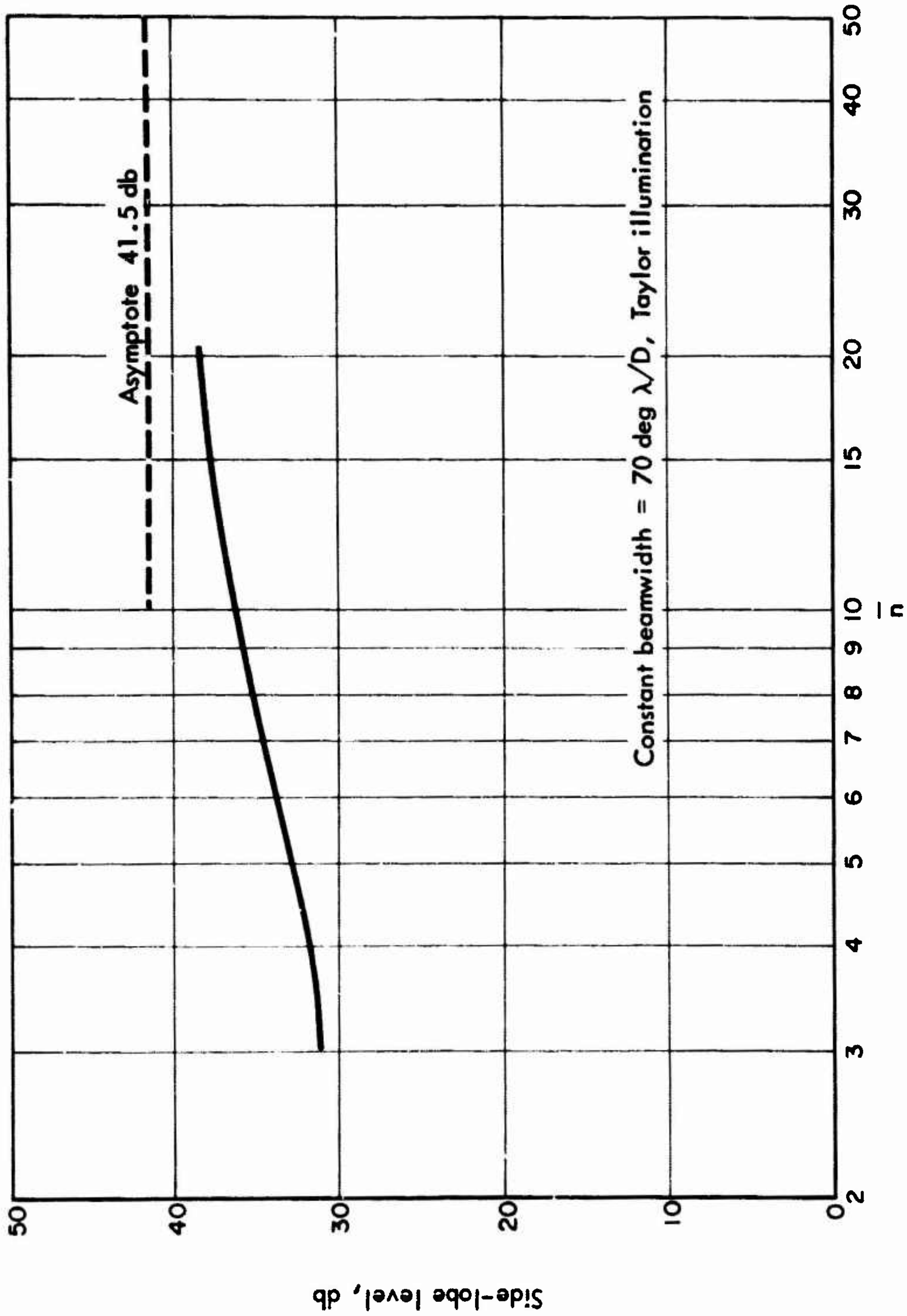

FIGURE 2 Variation of Design Side Lobe of $\bar{n}$

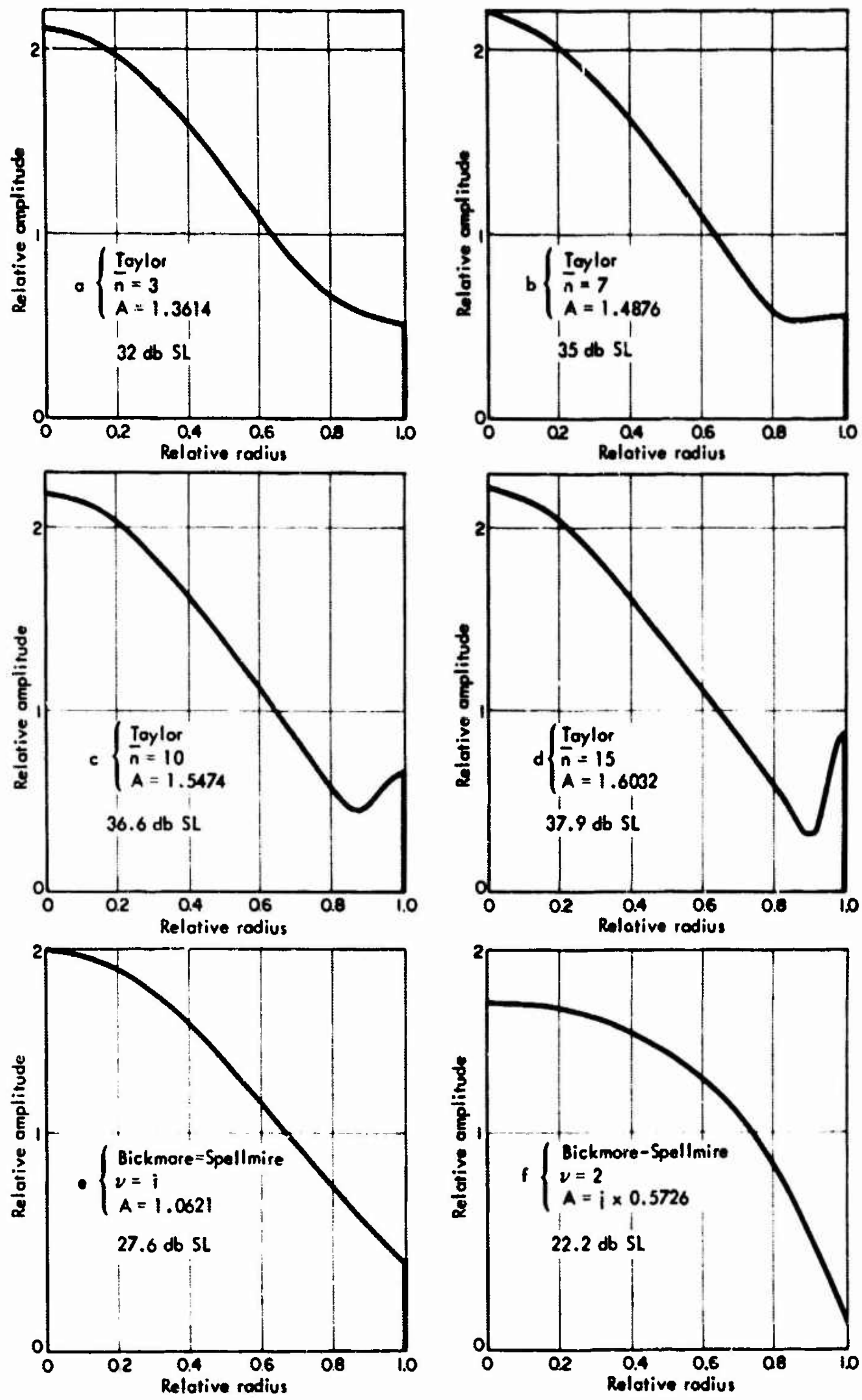

FIGURE 3 Various Aperture Illuminations Yielding a Beamwidth of 70 deg λ/D

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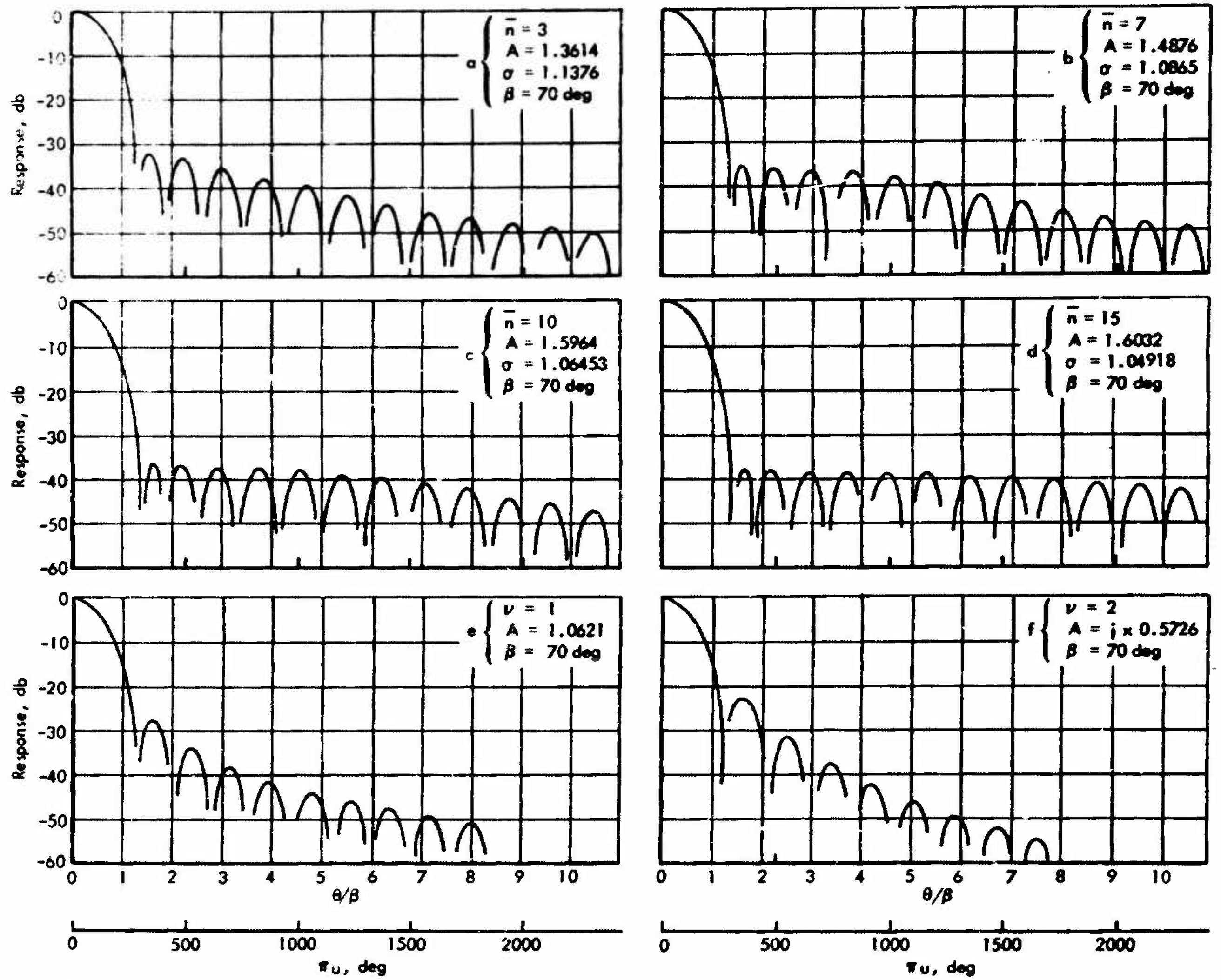

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FIGURE 4 Various Radiation Patterns having a Beamwidth of 70 deg λ/D

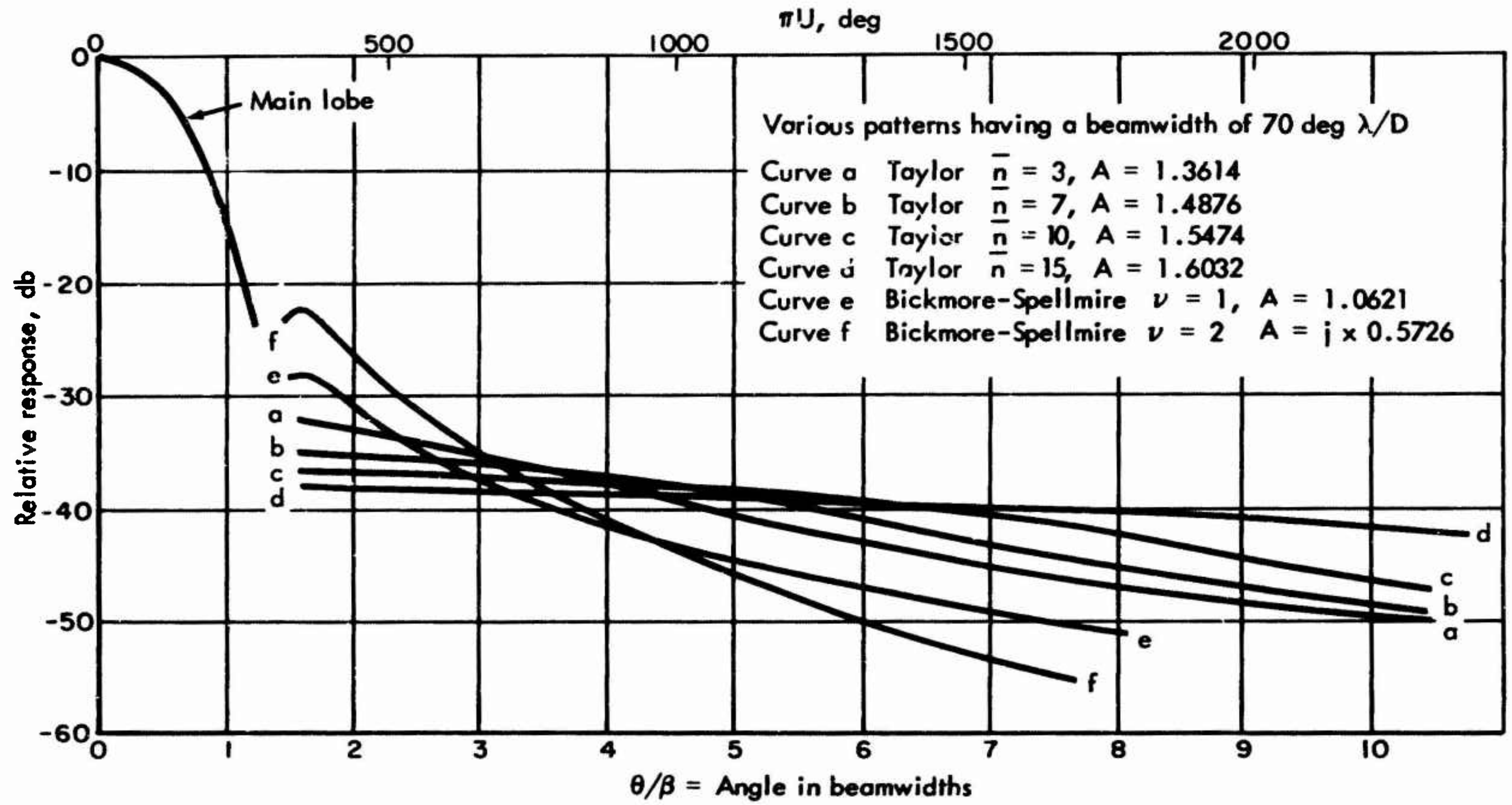

FIGURE 5 Side-Lobe Envelopes

R9-7 67-5

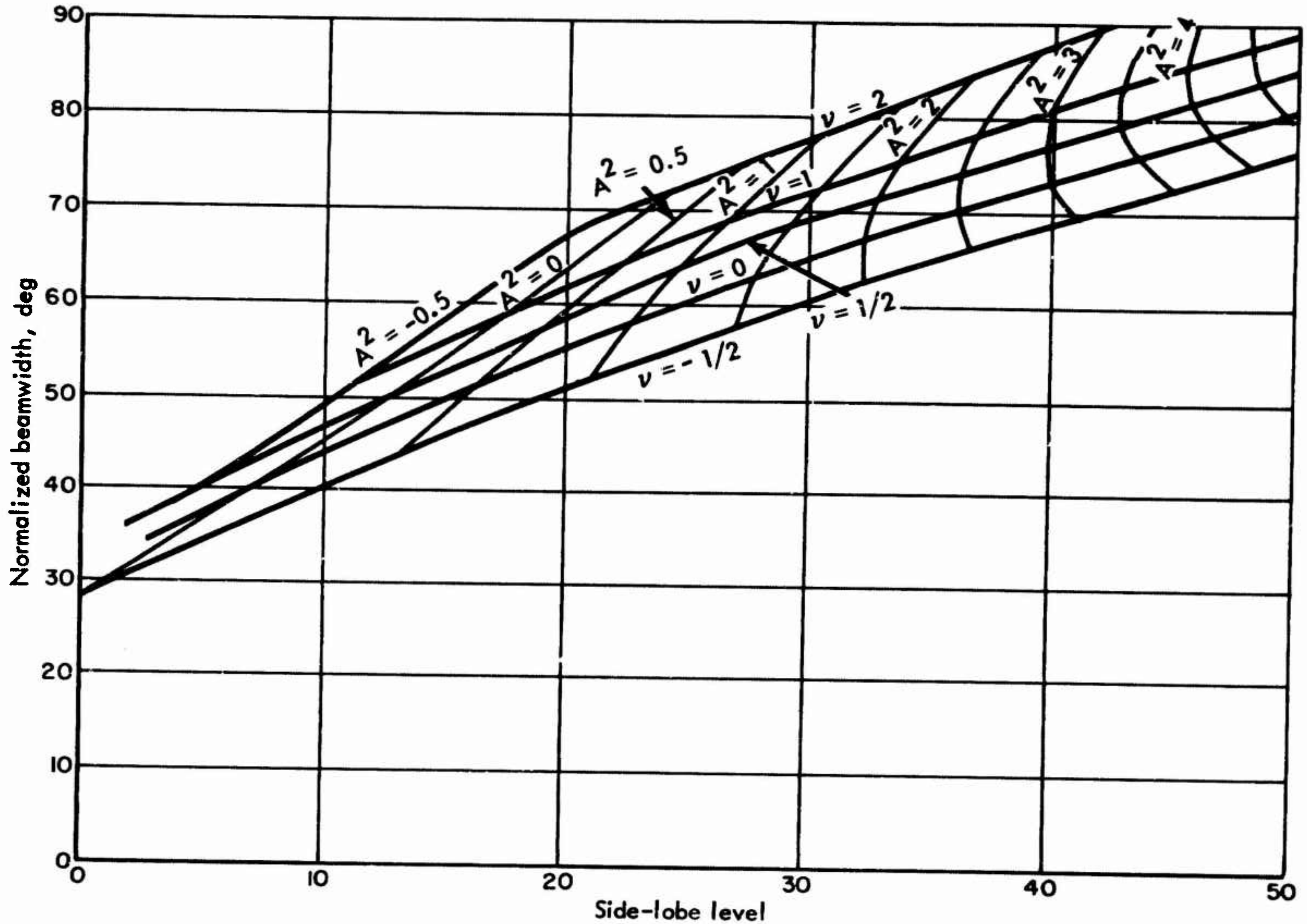

FIGURE 6 Beamwidth Versus Peak Side-Lobe Level, Bickmore-Spellmire Patterns

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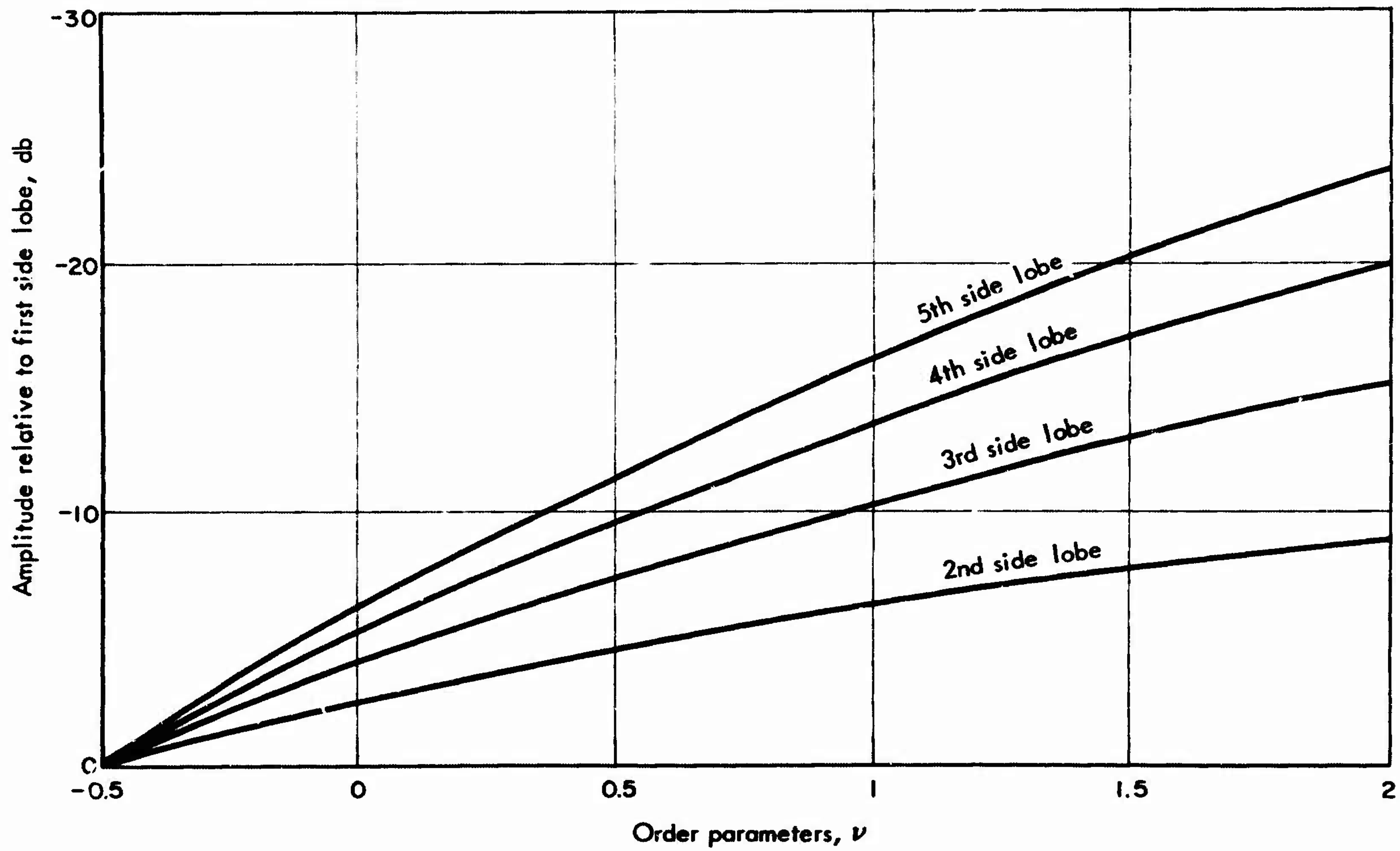

FIGURE 7 Amplitude of Remote Side Lobes, Bickmore-Spellmire Patterns

R9-7-67-7

REFERENCES

1. T.T. Taylor, "Design of Circular Apertures for Narrow Beamwidth and Low Side lobes," IRE Trans. on Antennae and Propagation, vol. Ap-8, pp. 17-22; January 1960.
2. C.L. Dolph, "A Current Distribution for Broadside Arrays Which Optimizes the Relationship Between Beamwidth and Side-lobe Level," Proc. IRE, 34, pp. 335-348, June 1946.
3. R.W. Bickmore and R.J. Spellmire, "A Two-Parameter Family of Line Sources," Hughes Aircraft Co., Culver City, Calif. Tech. Memo No. 595; October 1956.
4. Jahnke and Emde, "Tables of Functions," Dover, New York, 4th ed., 1945, pp. 180-188.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Institute for Defense Analyses		2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP None
3. REPORT TITLE Desirable Illuminations for Circular Aperture Arrays		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Research Paper P-351 December 1967		
5. AUTHOR(S) (First name, middle initial, last name) Warren D. White		
6. REPORT DATE December 1967	7a. TOTAL NO. OF PAGES 19	7b. NO. OF REFS 4
8a. CONTRACT OR GRANT NO. DAHC15 67 C 0011 b. PROJECT NO. ARPA Assignment 5 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) P-351 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) NA	
10. DISTRIBUTION STATEMENT This document has been approved for public release and sale; its distribution is unlimited.		
11. SUPPLEMENTARY NOTES NA	12. SPONSORING MILITARY ACTIVITY NA	

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